THE LEAST ACTION PRINCIPLE
AND
THE SPIN OF GALAXIES IN THE LOCAL GROUP

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Abstract

Using Peebles' least action principle, we determine trajectories for the galaxies in the Local Group and the more massive galaxies in the Local Neighbourhood. We deduce the resulting angular momentum for the whole of the Local Group and study the tidal force acting on the Local Group and its galaxies. Although Andromeda and the Milky Way dominate the tidal force acting on each other during the present epoch, we show that there is a transition time at $z \approx 1$ before which the tidal force is dominated by galaxies outside the Local Group in each case. This shows that the Local Group can not be considered as an isolated system as far as the tidal forces are concerned. We integrate the tidal torques acting on the Milky Way and Andromeda and derive their spin angular momenta, obtaining results which are comparable with observation.

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1. Introduction.

One of the problems facing any theory of galaxy formation is to explain how proto-galaxies can acquire spin angular momentum as they emerge from a non-rotating homogeneous distribution of gas. One of the early solutions was presented by Hoyle(1949), who suggested that the spin of a galaxy might result from tidal interactions with its neighbours. Hoyle's analysis was set against the background of a steady state universe, but the idea extends to an expanding universe scenario. It has been shown that tidal interactions can produce galaxies with the rotations of the right order of magnitude (Peebles 1969, White 1984, Barnes & Efstathiou 1987), although results beyond this remain elusive.

A major problem is that in order to calculate the torque on a given galaxy, its history and the histories of its neighbours must be known. Although we have relatively accurate positions for galaxies in the vicinity of the Local Group, we have only the radial component of their velocities. It is therefore very hard to trace their orbits back in time. Moreover, if this is attempted, the uncertainty in the velocities increases without bound due to unstable solutions of the equations of motion. A small error in the velocity now would translate into an enormous difference in position in the early universe. Thus it seems impossible to trace back galaxies using only the observational data.

Peebles (1989,1990) proposed a method he calls the least action principle, which he used to find complete trajectories for Local Group galaxies. The idea is to assume that galaxies growing out of small density perturbations in the early universe will have negligible peculiar velocities with respect to the Hubble flow. Using this as one boundary condition and the present positions of the galaxies as the other, trial orbits are iteratively varied so as to minimize the action. The method has been criticised since the galaxies are treated as point particles throughout their history, even though the size of the galaxies must be comparable to their separation at early times. However, the least action principle leaves the final velocities of the galaxies unconstrained, and its ability to reproduce the observed radial velocities remains a powerful test of the validity of the trajectories. For the Local Group galaxies, Peebles has obtained remarkable agreement between the observed radial velocities and those calculated from the least action principle.

Supplied with a possible history for all members of the Local Group we are able to evaluate the tidal torques acting on a particular galaxy at any point along its orbit. Using a suitable model for the evolution of the protogalaxy we can integrate the torque and investigate the acquisition of the galaxy's spin angular momentum. The magnitude of the angular momentum will strongly depend on the details on the model. However its direction should not be as sensitive. Thus we will primarily restrict our results to explaining the direction of the spin axis of the Milky Way and Andromeda.

Gott and Thuan (1978) have studied the angular momentum of members of the Local Group assuming it was isolated. They observed that M31 and the Galaxy lie nearly in each other's plane and that their spins are opposite as measured along the line of sight joining the two galaxies. This strongly suggests that the spin arose through tidal interaction. Assuming that M31 and the Galaxy are effectively isolated, they deduced that the original line of sight connecting the two galaxies must be perpendicular to the spin angular momentum vectors of both M31 and the Galaxy. This gives two possible trajectories. We will compare them with the ones given by the least action principle of Peebles (1989 and 1990). We will also test the hypothesis that these two galaxies could be thought as isolated. We will compare our results with those of Raychaudhury and Lynden-Bell (1989) who have studied the influence of external galaxies on the timing argument of Khan and Woltjer and proposed possible orbits for M31 and the Galaxy.

In the next section, we briefly review the least action method and determine a set of trajectories for galaxies in the Local Group and dominant galaxies in the Local Neighbourhood. In section 3 we calculate the orbital angular momentum of the Local Group and compare this with the results from Raychaudhury and Lynden-Bell(1989) as well as its implications for the analysis by Gott and Thuan (1978). We present the tidal interaction picture in section 4 and describe our adopted model of galaxy evolution. Finally, our results are presented in section 5 and we draw conclusions in section 6.

2. Trajectories for Local Group galaxies.

In order to determine a set of trajectories for Local Group galaxies, we follow the least action principle described by Peebles (1989 & 1990, hereafter P1 and P2). However, in order to study the torque acting on galaxies in the Local Group, we must also consider the effect of the dominant galaxies and groups in the Local Neighbourhood.

2a. The Least Action Principle.

Peebles' least action principle selects a set of classical trajectories for a group of galaxies (point masses) which are interacting through gravity, against the background of an expanding universe model. This method differs from the usual application of the least action principle in that boundary conditions are applied to the beginning and end of each trajectory. The trajectories are constrained such that

$$\delta \mathbf{x}_i = 0 \ \text{ at } \ t = t_0, \qquad a^2 d\mathbf{x}_i/dt \to 0 \ \text{ at } \ a \to 0 \tag{2.1}$$

where a is the scale factor of the universe and $\mathbf{x}_i(a)$ is the trajectory of the ith galaxy in comoving coordinates. That is, the galaxies are fixed at their present positions at the present epoch, and their peculiar velocities vanish as we approach the Big Bang. Trial trajectories for the set of galaxies are adjusted in order to find a stationary point in the action.

A $k = 0$ universe with a possible cosmological constant is assumed, thus

$$H dt = \frac{a^{1/2} da}{F^{1/2}}, \tag{2.2}$$

where $F = (\Omega + (1 - \Omega)a^3)$, H is the present Hubble constant and Ω is the density parameter. Following Peebles P2, the action for particles moving in such a universe is,

$$S = \int_0^{t_o} \Big[\sum \frac{m_i a^2}{2} (\frac{d\mathbf{x}_i}{dt})^2 + \frac{G}{a} \sum_{i \neq j} \frac{m_i m_j}{|\mathbf{x}_i - \mathbf{x}_j|} + \frac{2}{3}\pi G \rho_b a^2 \sum m_i \mathbf{x}_i^2\Big] \tag{2.3}$$

from which we can deduce the equation of motion,

$$a^{1/2} \frac{d}{da} a^{3/2} \frac{d\mathbf{x}_i}{da} + \frac{3(1 - \Omega)a^4}{2F} \frac{d\mathbf{x}_i}{da} = \frac{\Omega}{2F} [\mathbf{x}_i + \frac{R_0^3}{M_T} \sum_j \frac{m_j(\mathbf{x}_j - \mathbf{x}_i)}{|\mathbf{x}_j - \mathbf{x}_i|^3}]. \tag{2.4}$$

Here R_0 is the radius of a sphere which would enclose a homogeneous distribution of the total mass M_T of the group of galaxies considered, $R_0 \equiv M_T(\frac{4}{3}\pi\rho_b^0)^{-1}$.

It is very hard to have exact analytic solutions for the coupled system of equations (2.4). However Peebles succeeded in obtaining approximate solutions using trial functions of the form

$$\mathbf{x}_i(a) = \mathbf{x}_i^o + \sum_n \mathbf{C}_i^n f_n(a) \tag{2.5}$$

where $\mathbf{x}_i^o$ are the present positions of the galaxies and the f_n are a restricted set of 'Fourier modes' chosen to satisfy the boundary conditions (2.1). The classical solutions are obtained by introducing $\mathbf{x}_i(a)$ in the action and iteratively modifying the coefficient $\mathbf{C}_i^n$ to obtain a stationary action. In this paper we take $f_n = a^n(1 - a)$ for $n = 0, \ldots, 4$. As Peebles did, we verify that the least action solutions are good approximations to real solutions by evolving the classical equations of motion starting with the initial positions and velocities derived from the least action solutions at $z = 60$.

2b. Trajectories of Dominant Quadrupole Galaxies.

Peebles determined least action solutions for eight members of the Local Group in P1 and also studied the influence from the Maffei and Sculptor groups in P2. Here we will use the same Local Group members but also include five galaxies/groups in the local neighbourhood which contribute a significant torque on the Local Group. We adopt Peebles preferred universe from P2, $H = 75$ km s^{-1} Mpc^{-1} and $\Omega = 0.1$. This will permit us to study the influence of galaxies external to the Local Group in the past and examine their effect on the spin of the galaxies.

Raychaudhury and Lynden-Bell (1989, hereafter RL) studied the effect of nearby galaxies on the quadrupolar component of the gravitational field at the barycentre of the Local Group. They showed that the galaxies with the largest tidal influence on the Local Group lie within a radius of 7 Mpc. We follow their estimates for the distances to the most important nearby galaxies, but represent the Sculptor group (NGC 55, 253, 247, 7793) as a single point. Unlike RL, we also include the Maffei galaxies at a distance of 3.6 Mpc despite their low galactic latitude (b $=-1°$) and the lack of any independent distance estimates. Buta and McCall (1983) estimate a total extinction of at least five magnitudes to Maffei I. However, there remains considerable uncertainty over the distance to the Maffei group, and its corresponding mass. Similar concerns surround the distance to IC 342 except that internal absorption is the main problem in this case. Freedman and Madore (1992) estimate a total extinction of at least 2.2 magnitudes to IC 342.

Our final set of 13 galaxies is shown in table 1. Once again, we take the lead from Peebles, P1, when determining galaxy masses and set the masses to simplified ratios of the mass of M31. The Milky Way is taken to be 70% of mass of M31 whilst the others galaxies have relative masses shown in table 1 determined from a comparison of their corrected absolute blue magnitudes, based on the assumption that mass traces light. The total mass is fixed by adjusting the mass of M31 such that the least action trajectories reproduce the observed radial velocity of M31, $v_{M31} = -123$ km s^{-1}. The resulting mass of M31 is $M_{M31} = 10.8 \times 10^{12} M_\odot$ which corresponds to a mass to light ratio of $M/L_B \approx 240$.

2c. Comparison of the Trajectories.

After minimizing to find a stationary point in the action, we obtain the trajectories shown in figure 1. The parameters used to obtain trajectories of figure 1 give a universe of 17 billions years. In comoving coordinate, we can see that M31 and the MW start from a distance between each other comparable with the distance to galaxies outside the Local Group. Thus in the initial state of the universe the gravitational force on MW or M31 is dominated by galaxies outside the Local Group. This is true until the scale factor of the universe a was half the present value. Therefore, before this epoch, it is not possible to regard the effect of external galaxies as perturbations on the motion of Local Group members.

From the LAP trajectories we can also deduce the maximum distance between M31 and the MW which was $2.4 Mpc$ at $a \sim 0.4$. The mass of M31 has been adjusted so that the projection of the velocity on the line of sight is $-123 km/s$. However the LAP trajectories permit us to calculate the other components of a galaxy's velocity. We find a velocity for M31 of $436 km/s$ in the direction ($l = 226°$, $b = 8°$). The proper motion of M31 is 0.6 km s^{-1} kpc^{-1} (1.26×10^{-4} arcsec yr^{-1}), which is twice the value of the prediction made by RL, but still below observational limits. The components on the sky are $\dot{l} = 1.26 \times 10^{-4}$ arcsec yr^{-1} and $\dot{b} = 4.8 \times 10^{-6}$ arcsec yr^{-1}. This perpendicular component of the velocity gives rise to a large angular momentum for the Local Group. Since the proper motion of M31 is more significant than its line of sight velocity, we need a larger mass for M31 and MW than is deduced using in the timing argument of Kahn and Woltjer (1959).

Comparing the trajectories of figure 1 with those obtained by Peebles, we can see that adding more galaxies outside the Local Group leads to an increase in the initial comoving separation of the Milky Way and Andromeda. Since the external galaxies tend to pull the Milky Way and Andromeda apart, we find that we need to increase the total mass of the system in order to obtain a radial velocity of -123 km s^{-1} for Andromeda. A more massive system has a larger value of R_0, which explains the increased separation. We have also noted that the addition of these external galaxies degrades the agreement for the line of sight velocity of Local Group members that Peebles had obtained for $\Omega = 0.1$ and $H = 75$. In our case a higher value of Ω may give better results. However this does not change the qualitative behaviour of the LAP trajectories, we will therefore retain Peeble's values of H and Ω.

3. Orbital Angular Momentum in the Local Group.

Gott and Thuan (1978) have attempted to calculate the relative initial position of the Milky Way and Andromeda assuming that they form an isolated system which conserves angular momentum (spin and orbital). They also assumed, as we do, that the spin of these galaxies would arise from mutual tidal interaction. From these two premises they concluded that the original direction between these galaxies is perpendicular to the direction of their spins. This gave the result that the original direction of Andromeda with respect to the Milky Way must have been either $l = 152°, b = 0°$ or $l = 332°, b = 0°$.

However, if we examine the relative velocity of M31 and the Milky Way using our trajectories, the magnitude of the angular momentum of the pair with respect to their common centre of mass is $8 \times 10^{77} g\, cm^2/s$. This is much larger than their individual spin angular momenta which is of the order of $10^{74} g\, cm^2/s$. Thus the least action principle implies that the Local Group can not be thought of as a tidally isolated system.

Recently, Raychaudhury and Lynden-Bell (1989) showed that there is an appreciable quadrupole moment acting upon the Local Group even at the present epoch. Assuming that the eigendirections of the quadrupole moment did not substantially change direction and that the tidal force was only a small perturbation they calculated possible trajectories for Andromeda using the equation

$$\ddot{r^{\alpha}} - GM_t \frac{r^{\alpha}}{|\mathbf{r}|^3} = \sum_{\beta} Q^{\alpha\beta} r^{\beta}. \tag{3.1}$$

$\mathbf{r}$ is the separation vector between the Milky Way and Andromeda, M_t their total mass and $\sum_{\beta} Q^{\alpha\beta} r^{\beta}$ is the projection of the quadrupole component of the gravitational field in the radial direction. This equation assumes that the tidal force $Q \cdot r$ is weak compared to the inverse square law. They obtained various trajectories by assuming different time dependencies of the eigenvalues of the quadrupole moment. However, using the trajectories from section 2, we can calculate the quadrupole moment acting upon the Local Group as a function of the scale size of the universe. These are presented for $a = 0.1$, $a = 0.5$ and $a = 1.0$ in figure 2 where the projection of the quadrupole on the line of sight is illustrated. Table 2 summarizes the eigenvalues and eigendirections of $Q^{\alpha\beta}$ in each case. We can see that the eigendirections have changed since the Big Bang and that might have affected the tidal force. In figure 3 we present the trajectory of Andromeda obtained by Raychaudhury and Lynden-Bell and compare it with the one from the least action principle. They are qualitatively very similar. However our trajectory for $M31$ is longer. This is partly a result of using different values for Ω and h but also due to the fact that the tidal force due to local neighbourhood galaxies is not a perturbation in the very early universe and thus the equation used by Raychaudhury and Lynden-Bell is invalid.

4. Quadrupole—quadrupole Interaction.

Hoyle (1949) has suggested that the spin of a galaxy might result from the tidal interaction with neighbouring galaxies. Supplied with our set of trajectories for galaxies in the local neighbourhood, we can concentrate on any individual galaxy and determine the tidal torque acting upon it throughout its history. Using a model, which represents the selected galaxy as a rigid spheroid, we use the quadrupole-quadrupole interaction to numerically integrate the tidal torque acting on the selected galaxy and determine its final spin angular momentum.

The change in spin angular momentum is given by the torque

$$\Gamma_\alpha = \frac{dI_{\alpha\beta}\omega_\beta}{dt}. \tag{4.1}$$

where $I_{\alpha\beta}$ is the moment of inertia tensor of the selected galaxy, ω its frequency of rotation.

The torque due to the gravitational field generated by the distribution of point mass galaxies surrounding the selected galaxy is,

$$\Gamma_\alpha = \frac{1}{3}\sum_{\beta,\gamma} \epsilon_{\alpha\beta\gamma} q_{\beta\delta} Q_{\delta\gamma} \tag{4.2}$$

where

$$Q_{\delta\gamma} = \sum_i \frac{GM_i}{|\mathbf{x}-\mathbf{x}_i|^3}\Big[\frac{3(\mathbf{x}-\mathbf{x}_i)_\delta(\mathbf{x}-\mathbf{x}_i)_\gamma}{|\mathbf{x}-\mathbf{x}_i|^2} - \delta_{\delta\gamma}\Big] \tag{4.3}$$

is the quadrupole component of the gravitational field and $q_{\beta\delta}$ is the quadrupole moment of the selected galaxy.

4a. The Galaxy Model.

In order to calculate the spin of the selected galaxy, we have to know its inertia and quadrupole moments. Since the dark halo is the dominant component of a galaxy for gravitational interactions, we model the galaxies as spheroids. In fact in this paper the galaxy spheroids are rigid, which can only be approximation since in the absence of dissipation, protogalaxies which acquire their spin through gravitational interactions must have zero vorticity as no vorticity is observed in the microwave background. However, tidal forces are linear with distance from the center of mass and thus induce a state of approximate solid body rotation (Gott 1975), so this seems to be a reasonable approximation.

The quadrupole moment of the selected galaxy is,

$$q_{ij} = \frac{m}{5} R^2 (1 - \epsilon^2) diag(1, 1, -2) \tag{4.4}$$

in a frame defined by the axis of symmetry of the spheroid. The five undetermined parameters are the mass m of the galaxy, the length of the semi-major axis R, the eccentricity ϵ of the ellipse of revolution and two angles specifying the direction of the major axis.

In the gravitational instability picture small density perturbation of sufficient magnitude expand to a maximum size R_m and recollapse. In an $\Omega = 1$ universe the radius of the proto-galaxy obeys the equation

$$R = \frac{R_m}{2}(1 - \cos\eta),$$

$$a = \frac{a_{\rm col}}{(\pi)^{3/2}}(\eta - \sin\eta)^{2/3} \tag{4.5}$$

where $a_{\rm col}$ is the scale factor of the universe when the galaxy starts to recollapse. There is the following relation between R_m and $a_{\rm col}$

$$a_{\rm col} \approx \left(\frac{H^2\pi^2}{2GM}\right)^{1/3} R_m. \tag{4.6}$$

R_m remains a free parameter and is related to the time at which galaxies form. Binney and Silk have also shown that the eccentricity initially varies proportional to the scale factor of the universe, but rapidly becomes a quadratic variation. Here we will simply model the evolution of the eccentricity of a galaxy spheroid by,

$$\begin{aligned} \epsilon &= 1 - \epsilon_m (a/a_{\rm col})^2 \qquad && a < a_{\rm col} \\ &= 1 - \epsilon_m && a > a_{\rm col} \end{aligned} \tag{4.7}$$

We will present results for the case of $\epsilon_m = 0.81$ but we have also examined other values, all of which give a qualitatively similar picture. A more elongated spheroid has a stronger coupling to the tidal field, resulting in a larger spin angular momentum. Since the eccentricity of the galactic halo is essentially unknown, we do not feel that we can accurately predict the magnitude of the spin of any particular galaxy, but the direction of the spin axis is less sensitive to the shape of the galaxy model and so provides a better indicator.

4b. Initial Orientation of a Proto-galaxy.

The remaining free parameter of the model is the initial orientation of the major axis of the selected galaxy. We find that the orientation of the final spin axis is very sensitive to this parameter. This can be seen in figure 4 which plots the direction of the final spin axis calculated for the Milky Way and Andromeda as a function of the orientation of their major axes at $a = 0.1$. Successive evaluations were made for initial orientations based on a grid spaced by $10°$ in l and b, assuming $a_{\rm col} = 0.5$. We plot the projection of the unit spin vector of the model onto the unit vector of the observed spin. The lightest contour level corresponds to an angle, $\theta \leq 15°$, between the two vectors. The contours increase by $15°$ intervals up to $\theta = 90°$, with the final contour at $\theta = -45°$. The symmetry comes from the bi-polar symmetry of the galaxy spheroid model.

By definition the direction of the observed spin for the Milky Way is,

$$\hat{\mathbf{J}}_{MW} : b = -90°,$$

whilst for Andromeda we adopt,

$$\hat{\mathbf{J}}_{M31} : l = 242°, b = -30°$$

quoted by Gott and Thuan. However, these spin axes are determined from the rotation of the luminous matter in the galaxy, whereas our model more accurately represents the the dominant mass and therefore the dark halo component of the galaxies. Kuijken (1991) modeled the disc warps resulting from the misalignment between the luminous disc and the halo of a spiral galaxy. The Galaxy is observed to have a moderate disc warp, and he estimates that there could be a discrepancy as large as $30°$ between the observed spin axis and the actual spin of the halo.

From figure 4a we can see that a random choice for the initial orientation of the Milky Way would have approximately a 1.7% chance of producing a final spin within $15°$ of the observed direction. The a priori chance of selecting a vector within $15°$ of another is 3.4%. Conversely, the similar probability for Andromeda is $\sim 5.8\%$.

However, we do not have an entirely free hand when selecting the initial orientation for a galaxy. Binney and Silk (1979) have shown that a first order effect of tidal interaction is to produce an anisotropy in the density perturbation of a protogalaxy. The anisotropy is aligned so as to minimise the torque on the galaxy. This inhibits the early rotation of the galaxy, but gives a preferred direction for the initial orientation of its major axis. As the matter distribution around the galaxy changes, there will be a change in the direction of the local axis of the tidal field which will induce the galaxy to spin. In figure 5 we have plotted the magnitude of the tidal field $\propto m/d^3$ for the Galaxy and Andromeda as a function of the scale size of the universe. At the present epoch, Andromeda provides the dominant contribution to the quadrupole acting on the Milky Way and vice versa. However, at early times the quadrupole moment acting on them both is dominated by galaxies external to the Local Group. In both cases the transition in the quadrupole occurs at a redshift of between $z = 0.5$ and $z = 1$. From qualitative arguments we see that if these galaxies collapse much before this transition time ($a_{\rm col} < 0.4$), very little angular momentum will be transferred to them.

5. Results.

So far we have not fixed the value of $a_{\rm col}$. At the time of writing, there is little observational evidence for a population of proto-galaxies at $z < 1$, which argues against $a_{\rm col}$ being much larger than 0.5. On the other hand, we find that it is comparatively difficult to get the galaxy to spin at all if $a_{\rm col} < 0.3$. This is easy to see qualitatively. With the Milky Way pointing towards IC 342 initially, there is little change in the direction of the eigenvectors of $Q_{\delta\gamma}$ until $a = 0.3$. Then as the separation between the Milky Way and Andromeda decreases, Andromeda comes to dominate the quadrupole moment and the major axis of the Milky Way swings round to point towards Andromeda. After $a_{\rm col}$, $q_{\rm MW}$ becomes small and the galaxy effectively decouples from the tidal field, rotating faster as it collapses, but with little change in the direction of the spin axis. Hence $a_{\rm col}$ should lie in the range 0.3 to 0.5. We have opted for $a_{\rm col} = 0.5$ since this results in a larger spin angular momentum.

The position of the galaxy which dominates the quadrupole moment at $a = 0.1$ provide a preferred direction for the initial orientation of the selected galaxy. From figure 5 we can see that the preferred choice for the initial orientation of the Milky Way is towards IC 342 ($l = 109°$, $b = 06°$ at $a = 0.1$). This gives disappointing results, with the final spin axis in the direction ($l = 193°$, $b = -12°$) – approximately 80° away from the observed direction.

We have investigated the possibility that initial quadrupole is not dominated by IC342 but rather by N5128. This might be a possibility if the mass of $N5128$ has been underestimated by a factor of a few percent or the distance by a factor of $1.2^{1/3}$. Using NGC 5128 to define the initial alignment, leaves the galaxy rotating with the spin axis oriented towards ($l = 327°$, $b = -74°$), within 16° of the observed direction. The resulting angular momentum is $5.2 \times 10^{74} gcm^2/s$. A similar situation exists with Andromeda. In this case the Maffei group would appear to dominate the early quadrupole. Aligning Andromeda towards it gives a spin in the direction of ($l = 44.1$, $b = -0.6$). Pointing towards N5128 is little better, but the M81 group yields the result $\mathbf{J}_{M31}$: ($l = 238°$, $b = -38°$). This is about 9° away from the observed direction, so once again we are in remarkably good agreement with the observations. In this case the obtained magnitude is $1.0 \times 10^{75} g\ cm^2/s$.

For completeness, figure 4a shows the direction from the Milky Way to the other galaxies in the sample at $a = 0.1$ and figure 4b shows the positions relative to Andromeda. In both cases, only one galaxy lies within the top contour which corresponds to a final spin less than 15° from the observed direction.

6. Conclusion.

We have investigated the angular momentum of the Local Group and its largest galaxies, the Milky Way and Andromeda, using the least action principle of Peebles. We have augmented the sample of galaxies to include galaxies from the Local Neighbourhood which contribute significantly to the present torque. We have found that the Local Group has a large angular momentum of order $8 \times 10^{77} gcm^2/s$ in the direction ($l = 75°$,$b = 75°$). This implies that the Local Group cannot be thought of as tidally isolated, and the total angular momentum in the Local Group is not conserved.

We have also investigated the angular momentum of the Milky Way and Andromeda. We have found that there is a transition time ($z \approx 1$) when the tidal forces on them is dominated by Local Neighbourhood galaxies and thereafter Local Group ones. The least action trajectories were able to reproduce the direction of the spin of these galaxies within 15°. In principle, once the trajectories of the members of the Local Group are known, we are in a position to integrate the torque acting on an individual galaxy and determine the final amplitude of the angular momentum. This might provide another test of the least action trajectories, but in practice we find that the results are more sensitive to the details of the model used to represent the evolution of the protogalaxy.

Acknowledgments. We would like to thank D. Lynden-Bell for useful comments and S. Raychaudhury who provided us with a his catalogue of nearby galaxies derived from the Kraan-Korteweg catalogue.

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Name	l	b	R (Mpc)	v_{CLG} (km s^{-1})	M_r	$m_B^{0,i}$	type
local group							
Milky Way	—	—	0.0	0.0	0.700	—	S
NGC 6822	25.3	-18.4	0.5	-179.8	0.025	9.35	I
M 31	121.2	-21.6	0.7	-539.7	1.000	4.38	S
IC 1613	129.7	-60.6	0.7	-348.3	0.025	9.99	I
WLM	75.9	-73.7	0.9	-198.0	0.025	11.04	IB
Sextans AB	246.2	39.9	1.3	113.4	0.025	11.87	IB
NGC 3109	262.1	23.1	1.7	129.6	0.100	10.39	S
NGC 300	299.2	-79.4	2.2	96.9	0.100	8.70	S
local neighbourhood							
Sculptor	105.8	85.8	3.2	221.1	2.000	9.00	S
Maffei AB	136.4	-0.4	3.5	199.0	2.000	14.80	E/S
M 81	142.1	40.9	3.5	103.3	0.410	7.86	S
NGC 5128	309.5	19.4	4.9	307.8	1.500	7.89	L
IC 342	138.2	10.6	6.1	227.7	4.000	9.42	S

Table 1. The sample of nearby galaxies and their properties. The galaxies are divided into two sections, (i) Local Group galaxies studied by Peebles and (ii) other dominant galaxies up 7 Mpc from the Local Group. The columns are (a) Name, (b,c) present galactic coordinates, (d) distance from the galaxy, (e) present velocity with respect to the centre of the Local Group, (f) Mass relative to the mass of M31, (g) Corrected apparent blue magnitude, and (h) Hubble type.

Scale Factor	*Eigenvalues*	l	b
a=0.1	313	302	22
	-240	213	-0
	73.2	303	-67
a=0.5	3.24	126	-17
	0.96	317	-71
	-4.20	37	3
a=1.0	1.45	138	33
	0.29	132	-56
	-1.75	227	-3

Table 2. Eigenvalues and eigendirections of $Q^{\alpha\beta}$ evaluated at the barycentre of the Local Group for three different scale factors a. This shows the importance of the variation of the eigendirections of the quadrupole moment as a function of time. The eigenvalues are in units of $0.43 \times 10^{12} \mathrm{M}_{\odot}/\mathrm{Mpc}^3$.

Figure captions.

Figure 1. Least action trajectories for galaxies of Table 1. The x direction is towards Andromeda at $\alpha = 10°$, $\delta = 41°$, the y direction towards $\alpha = 100$, $\delta = 0°$ and the z direction $\alpha = 190°$, $\delta = 49°$.

Figure 2. History of the tidal force $Q.r$ due to galaxies external to the Local Group, projected on the sky viewed from the center of mass of the Local Group. The three projections correspond to $a = 0.1,\ 0.5,\ 1.0$ showing that the eigendirections have changed since the Big Bang. The eigenvalues and eigendirections are also shown in table 2.

Figure 3. The sky projection of the least action trajectory for M31 from this paper (the plain line) compared with the trajectory obtained by Raychaudhury and Lynden-Bell (the dotted line). The least action trajectory is slightly longer, partly due to a different choice of the density parameter Ω and Hubble constant H. It is also due to the fact that for the least action trajectories, it is not possible to assume that the quadrupole force from galaxies outside the Local Group is only a perturbation. In the early universe these later galaxies dominate the force on the Milky Way and Andromeda.

Figure 4a. The projection of the unit spin of the Milky Way onto the unit vector in the direction of the observed spin as a function of l,b of the orientation of the galaxy model.

Figure 4b. The corresponding result for Andromeda.

Figure 5. Quadrupole moment on the Galaxy and M31 in function of the radius of the universe and z. There is a transition time ($a \approx 0.5 - 0.7$) during which the dominant contribution to the quadrupole moment passes from galaxies outside the Local Group to those within.